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AFDELING ZUIVERE WISKUNDE
(DEPARTMENT OF PURE MATHEMATICS)

ZN 97/80 SEPTEMBER

A.E. BROUWER & R.M. WILSON

THE DECOMPOSITION OF GRAPHS INTO LADDER GRAPHS

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The decomposition of graphs into ladder graphs

by

A.E. Brouwer & R.M. Wilson

ABSTRACT. We Use Vizing's theorem

Recently decomposing graphs into isomorphic copies of graphs from a given collection has become a popular pastime - the main motivation being that a pairwise balanced design is nothing but a decomposition of a complete graph into complete graphs, and that the techniques for constructing designs also work in this situation. (For references see [1].)

Caro & Schönheim considered decomposition of graphs into copies of tK_2, the ladder graph with t edges and 2t vertices, and gave a complete solution of the problem for t = 2.

We proved their conjectures for general t by observing that they are a consequence of Vizing's theorem.
(Since people ask for a reference I (=AEB) write this note - I hope RMW will forgive me for attaching his name to it.)

Let G be a graph with n vertices and m edges, of maximum degree d. If G is decomposable into graphs tK_2 for some fixed t then clearly m/t is an integer and $d \leq m/t$.

Conversely, suppose that the edges of G can be coloured with m/t colours. Since the union of two colour classes is a graph with paths and circuits as components we can recolour it with two colours such that the two colours occur almost equally often. Repeating this process as long as necessary we obtain a colouring with m/t colours where each colour occurs t times. But this is nothing but a decomposition into graphs tK_2.

Finally recall that Vizing's theorem says that the edge-colouring number of a graph is either d or d + 1, and that in the latter case $m \geq (3d^2+6d-1)/8$ (cf. Wilson & Fiorini [2], thm. 5.1 and thm. 13.2).

Thus: G can be decomposed into graphs tK_2 if and only if (i) (m/t is integral) and (ii) ($d \leq m/t$) and not (iii) ($d = m/t$ and G is of class two).

In particular: the necessary conditions (i), (ii) suffice when G is bipartite (and hence also when G is a forest) and when $m \geq t(8t-5)/3$.

KEY WORDS & PHRASES: G-*design, decomposition, Vizing's theorem*

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[2] FIORINI, S. & R.J. WILSON, *Edge colourings of graphs*, Research Notes in Math. 16, Pitman, London etc., 1977.

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