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Mathematics as Technology: Mathematical Engineering

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Abstract: In 1956 the study of mathematical engineering, Wiskundig Ingenieursopleiding, was established as a separate subject in The Netherlands at the Delft Technische Hogeschool. Its designer, Reinier Timman, derived his ideas from experience with the most advanced technological research of the time, in aeronautics and shipbuilding. Surmounting traditional Applied Mathematics by the new style of mathematical modelling, he placed mathematical thought in a surprising technological perspective.
After 150 years of diametrical opposition to practice within the engineering disciplines, mathematics was presented as another engineering discipline itself. The pure science of mathematics was reintegrated in the broader stream of mathematical thought. The appeal to mathematical thought was now caught in a procedure, mathematical modelling. Being a procedure, modelling made mathematical thinking as such appear under a technological aspect.

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Introduction

In 1956 the study of mathematical engineering, Wiskundig Ingenieursopleiding, was established as a separate subject in the Netherlands at the Delft Technische Hogeschool. Its designer, the mathematician Reinier Timman (1917 - 1975), derived his ideas from experience with the most advanced technological research of the time, in aeronautics and shipbuilding. Surmounting traditional Applied Mathematics by the new style of mathematical modelling, he placed mathematical thought in a surprising technological perspective.
Timman's combined background in university mathematics and in technological research reflects the double roots of mathematical engineering. Indeed the new discipline signified a remarkable development in mathematics as well as in engineering, as emerges both from the general historical evolution of the two fields and from the details of the Delft events.

The engineering sciences have been founded on a mathematized worldview. The tension is innate between practice and theory, between practice-orientedness based on experience and the effort to make the immanent mathematics explicit. Now after 150 years of diametrical opposition to practice within the engineering disciplines, mathematics was presented as another engineering discipline itself. This apparent anomaly was telling for the state of technology and engineering, now counting mathematics among their ranks. For its part, mathematics restored relations with use, with the practical applications that hadn't been addressed by pure nor by applied mathematics since the rise of the latter. Moreover, mathematical modelling being a procedure, it established mathematical thought as a technology.
The details of the Delft story, to be related in sections 3 and 4, reveal quite the same pattern. Delft

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Simon Stevin 1548-1620
Picture from: *The Principle Works of Simon Stevin* ed. by E.J. Dijksterhuis, D.J. Struik e.a. Amsterdam: Swets & Zeitlinger, 1955-1966 (5 Vols).

mathematicians were traditionally involved in propaedeutic mathematics (service courses), whence their majority was reluctant to appreciate the new role their science could play. The Mechanical Engineering Department, on its part, challenged the mathematicians for the authority over the new specialization. Timman found the narrow passage, between his hesitating colleagues and the outside competitors, to the new training scheme.

1. Practitioners and engineers

Mathematics did play its role in the engineering disciplines all along, as an ideal way of thinking and as basic, propaedeutic, education. There is a traditional asymmetric connection. But even for the performance of mathematics and engineering at equal level and status there exists a precedent. They shared common ground in the art of the seventeenth century mathematical practitioners.
Stevin, more than anyone else Simon Stevin (1548 - 1620) was the engineering mathematician. Moreover, Stevin designed the program for the first state engineering school, which started in Leiden in the year 1600. Engineers were trained for the professions of land- surveying, wine-ruddering and fortification building. Lectures were given in the Dutch language, reason why the program is better known as Duytsche Mathematicque, Dutch Mathematics.
One may construct a succession of schools and programs via different Dutch universities and military academies in order to connect the present Technical Universities in the Netherlands with that very early beginning. It will indicate that the engineers, rather than the mathematicians, are to be considered the heirs of Stevin.
The important features to keep in mind are that lessons were in Dutch, a clear sign of vocational training, and that the subject was called Mathematics ("Mathematicque", and not even Stevin's Dutch term "wiskunde"). Its artisans were called mathematical practitioners, not mathematical engineers.

What then was new about mathematical engineering in the 1950's? Reinier Timman presented his notion of mathematical engineering in 1953, the study opened its courses in 1956. In a kindred but less specific way the German Technische Hochschulen allowed their mathematics students to obtain the degree of Engineer (Diplommathematiker) since 1942. The Japanese created a mathematical engineering branch in 1948 at the spot where aeronautics research had been banned. In the USA John Tukey proposed mathematical engineering in 1954, and he got his way at some institutes, while most stuck to a specialization of mechanical engineering. Thus, mathematics and engineering reached another merger by the middle of the twentieth century, and it was a novelty because neither had remained the same. The reason for the novelty was in the adventures of mathematics since 1600.
The work of the mathematical practitioner had dispersed into different branches of engineering, which by 1800 became engineering disciplines with their innate tension between mathematics and practice. *The least the proponents of mathematical engineering had to offer was thus a way to reconcile or to surmount that tension.* Also around 1800 mathematics was stripped of its practical branches and narrowed down to a discipline of pure science: a twin discipline of pure science, to be exact, including pure and applied mathematics. *The contact, then lost, with the world of practice would need to be restored by the proposed mathematical engineering.*

2. Emancipation of mathematics

In the days of Stevin there was no such thing as applied mathematics. There was full mathematics, the "Geheele Mathesis", covering the whole range from "geometrica practica" to geometry, from "arithmetica practica" to algebra. Within the wealth of this domain there was no place for an "application"-relation between one part and another. Mention was made of mixed mathematics; and around 1700 the term "applied mathematics" was used for the first time [Mulder 1990]. As a distinct discipline however Applied Mathematics reached full deployment only after 1800 as a companion to the budding Pure Mathematics.
Apparently, only apparently, against the tide of the first industrial revolution and the French revolution mathematics emancipated from Natural Philosophy. It freed itself from the perspective of usefulness and from the idea of finding its ultimate goal in astronomy and mechanics [Struik 1948: Chap VIII]. It even resigned from the post of highest authority of Enlightment rationality. It entered the nineteenth century as a self-conscious and self-reliant discipline, dismissing in the best tradition of Enlightment all outer authority. The resulting pure science involved both Pure and Applied Mathematics.

Mathematics itself drastically changed in the process, its foundations and its reasonings being purged from supposedly metaphysical and empirical elements. The emphasis shifted from result to method and proof. Emancipation meant that a more proper insight was gained in what mathematics is (Emancipation also implied that mathematics precipitated into a sequence of foundational crises).
And fences were raised to mark the limits. Inside were the mechanics and mathematics of Newton, Huygens and Lagrange as purified by the latter to rational mechanics and applied analysis. Lagrange litteraly set out to unmix mechanics, trying to avoid the infinite and to avoid Newton's supposedly empirical notion of force. Although he did not succeed completely, he did set standards for Applied Mathematics as Applied Analysis: Applied Mathematics was basically the study of particular cases of differential and integral equations in view of some situation outside mathematics, without however letting any further aspect of that situation interfere with the reasoning — i.e. proofs were to be mathematical proofs, no hybrid reasonings were allowed —, and the adequacy of the work would only be judged afterwards.
Typical of the tradition of applied mathematics since Lagrange is its Promethean impatience. This impatience is the ever reoccurring expectation that nothing will enhance *practical* applications more than a theoretical breakthrough. The naive and persistent denial of the very nature of engineering, implied in this impatience — the very assumption that technical innovation would as a rule arise from applying science; the very assumption that the problem of technically mediating theoretical insight might simply be discarded in special cases, whereas in reality the problem merely shifts —, was in effect to cause repeated controversy.
As can be guessed from the impatience, applied mathematics was included in the purified discipline of mathematics. Excluded were all the mixed parts of mathematics, most notably the legacy of Stevin and that of Condorcet: engineering mathematics and societal mathematics. Inside were Lagrange, Cauchy, Fourier and Navier; outside were Laplace, Quetelet, political arithmetic (economics) and the rising disciplines of engineering.

All these subjects for which just then the Enlightment had declared the "Esprit Géometrique", i.e. mathematical thought, most fruitful were now chased from the holy grounds. That was the division which took place around 1800, not between pure and applied mathematics but between mathematics strict and mathematical thought at large. Mathematicians left technical and social presumptions to others. Although when confronted, the mathematician will be tempted to take the "platonic" stance and pretend to capture the true essentials of life or reality. Hence the Promethean impatience.
The dividing line lay between Cauchy and Laplace, between the *Cours d'Analyse* and the *Essay philosophique.* As a result every higher education of engineers would since start off with mathematics (and physics). The mathematicians were hired not to present applicable, let alone practical, mathematics, but to demonstrate the mathematical way of thinking.
This *propaedeutic function* of mathematics was the paradoxical result of Enlightment. Through the propaedeutic function mathematics, "pure" or "applied", left hardly any tractable results in practical engineering, but it definitely tainted the world picture of the engineering sciences.

In itself propaedeutic mathematics developed into a somewhat dull and never controversial trade. It was only when it interfered with that other more palpable function, viz. actual application, that tensions would be raised fast and high. E.g. when at the German Technische Hochschulen mathematicians nourished too great presumptions, trying to sell pure geometry as applicable mathematics, this was one of the factors causing the Anti- Mathematische Bewegung in the 1890's [Hensel 1989].
Structural interference of the two functions of mathematics rose in the second quarter of the 20th century, as a matter of fact resulting from successful titration of mathematical thought in the engineering disciplines over the foregoing century [Klemm 1966], [Dresdener: passim].
Engineers started to want higher mathematics for application purposes. The awkward situation presented itself in the 1930's that engineers who surpassed the mathematicians in (applied) mathematics, demanded the maths professors in the Delft Technical University to be more application-oriented.

At first the mathematicians complacently clung to their propaedeutic function, claiming to their defense that all they had to convey was a way of thinking. The engineers, quite dissatified, pointed at descriptive geometry — an obsolete subject, to their judgement, the lessons and drawing practices of which took many hours the engineering student might have spent learning useful stuff, applicable maths.
Here of course is the irony of history at work. From the outset of the Ecole Polytechnique descriptive geometry was the pride of Gaspard Monge and his fellow mathematicians. Here was mathematics presented as a universal language of utmost clarity. By 1930, a good century later, this eternal good proved rather transient.

3. Delft

Let us now trace the story in closer detail, observing the events in Delft from the late thirties onwards. The Technical University, Technische Hogeschool, had a Department of General Sciences, including a considerable section of professors in mathematics. This department would not have students of its own and exclusively teach service courses for the technical departments (e.g. the Department of Electrical Engineering, of Mechanical Engineering — including Aeronautics and Shipbuilding —, and of Construction Engineering). The physicists had just then split off from the general department to create a (sub)Department of Technical Physics with its proper study for Physics Engineer. Meanwhile maintaining their propaedeutic function, the physics professors played an important double role.
From this setting, two simultanuous developments prepared the ground for the study of mathematical engineering. The propaedeutic function of mathematics came under assault, and the maths professors were forced into action (3.a). The subculture of scientifically oriented engineers, cultivating a new style of applying mathematics, gained recognition within the Delft community (3.b).

3.a. Delft propaedeutic mathematics

Desciptive geometry was one of the topics the Delft mathematicians clung to with all their might, their rather restricted might. But the relevant residue of descriptive geometry had long been fully absorbed by the Construction Engineering Department and the Mechanical Engineering Department in their own introductory courses: projective methods in construction statics and machine- part drawing.
One professor of Workshop Installation in the Mechanical Engineering Department caused a row in 1939 when he planned his machine-part-drawing practices and exams on the hours reserved for drawing practice in descriptive geometry. The deliberate defiance — the same professor had been active in debates with the mathematicians on related issues — was covered up as a misunderstanding. Still it did set things in motion. The Department of Technical Physics and those of Electrical and Mechanical Engineering shared the frustation of coping with the situation and acquiring their own expertise in vector- and matrix- calculus, in special differential equations, in probability- calculus. Such knowledge was available and elsewhere, at a few other locations, mathematicians did hold the expertise. Thornton C. Fry at Bell Labs and MIT, and Richard von Mises in Berlin had lead the way to new and quite different opportunities for the mathematician. Von Mises was founder and editor of *ZAMM, Zeitschrift für angewandte Mathematik und Mechanik* [Fry 1928; 1941; 1963], [Mises 1921].
Once the conflict had been triggered, they stood up and forced the Delft professors of mathematics to develop a more truthful answer.
In one meeting, early 1940, the mathematicians confronted themselves with the Department of Electrical Engineering. Representants of the latter claimed that the time of purely formative, i.e. thought educating, mathematics had passed; that the mathematicians could no longer afford to withdraw in the bullwark of the propaedeutic function; that the engineering disciplines had developed an explicit need for applications of mathematics. Bähler, one of the Electrical Engineering professors put it this way:

> "In technology there is a dynamic line, notably in applying mathematics. Therefore today's value of

mathematics is far more than formative" [Minutes 1940].

That was in 1940. Following the confrontation a committee on the subject was installed, but wartime prevented its recommendations from taking effect.

After the war was over, nothing had changed. Even a proposal to dissolve the whole section of mathematics was not adopted by the university board. Then again, a lot had changed. Changes in personnel had revitalised the section and the propaedeutic function of mathematics was restored in full pride. Education was intensified with "instructors" to supervise the practicing classes.
Although the two-person minority of professors favouring application oriented mathematics had meanwhile left Delft, attempting to realise their ideas elsewhere, another committee was installed in 1947: its commitment was to investigate the possibilities of an Engineering Degree in Mathematics.

Now that the propaedeutic function of mathematics was no longer under siege, there was no overt opposition to institutionalising the application function. However, it was nobody's zeal. Rather half-heartedly two options were submitted to the consideration of the Technical Departments:
1. a mathematical course of studies by way of specialization within the technical departments
2. a program in mathematical engineering within the section of mathematics.
As the choice was left to the others neither option was realized. The importance of the 1947 proposal was that the very idea of a study in Mathematical Engineering established itself in the minds of the maths professors — however vague the contours of the concept may have been.

3.b. Delft mechanical engineering
In a simultaneous effort of innovation the Department of Aeronautics, Shipbuilding and Mechanical Engineering installed a separate section of Mathematics and Mechanics by January 1949, thereby allowing a "theoretical" specialization for the engineers degree.
Thus far those holding the chairs in Applied Mechanics and in Aero- and Hydrodynamics had not been allowed to supervise students for the engineering degree. In the Delft engineering culture the work of the two holding these chairs, C.B. Biezeno (1888 - 1975, in office from 1914 to 1958) and J.M. Burgers (1895 - 1981, in office from 1918 to 1955) respectively, had been considered too theoretical; moreover by their strong programs in scientific research they were a local exception. These circumstances created a subculture of highly sophisticated research in the modern branches of Mechanical Engineering: the subculture of Biezeno. Biezeno's subculture was internationally recognized. The congress the young professors gathered in Delft in 1924 turned out to be the first in a continuing series [Biezeno/Burgers 1925]. Furthermore the subculture held close links with research in aspects of mining and railways, shipbuilding and most notably in auronautics.

Engineers in this field would typically deal with dynamic problems (as opposed to kinematic) involving essentially non-linear equations (such as fluid mechanics, in particular laminar flow, and elasticity, especially buckling problems) and they would solve these through sophisticated computation. Perhaps most characteristic of Biezeno's subculture was the approach of not avoiding the development of novel numerical methods taylored to the problem involved. As an outer characteristic, it may be mentioned that they belonged to the readership of *ZAMM, Zeitschrift für angewandte Mathematik und Mechanik.*

As late as in 1949 this subculture acquired due recognition at home, when the Department of Aeronautics, Shipbuilding and Mechanical Engineering allowed students to specialize in the "theoretical" branch. The recognition was eased by the general tendency — related to but distinct from the discussed specialization — to distinguish between a "research version" and a "business version" of the Delft study courses.
The Biezeno subculture now presented both part of the pressure upon the mathematics section to cultivate applications, and an appreciable competitor to the mathematicians' possible initiative. In fact,

the rise of the subject was an international phenomenon, and in several universities of technology the new applications of mathematics remained within the departments of Mechanics or Aeronautics.

4. TIMMAN

The National Aeronautical Laboratory in Amsterdam was closely linked to Biezeno's subculture. Here was one of these centres of daring computational effort. Flutter, buckling problems, fatigue problems, streamlining —the dynamical approach of all such matters was endeavoured from the thirties onwards, and through the war to reach a heigth in the 1950's. In 1946 a mathematician, a graduate form the University of Amsterdam, joined the Aeronautical Laboratory to work on the particular theme of flutter phenomena. This man was Reinier Timman. He had gained his doctors degree in Delft with Burgers in 1946 on theoretical flutter analysis. One year he substituted Burgers lecturing on theoretical aero- and hydrodynamics. At the laboratory he was very successful in working out his results through numerical analysis and computation. These excercises in numerical analysis moulded his ideas on modern "applied mathematics".
R. Timman was offered a chair in Delft in 1952 in the mathematics section on the preconcieved idea that he was going to realize a program for the subject of applied mathematics. So he did, and he called it mathematical engineering.
Timman's committee —of course another committee was installed— went straight for it. First asking the Mechanical Engineering and the other Departments what they were offering, it was easy for Timman to find the loophole for his own branch of studies.
The concept was fixed by 1953, and in 1956 the route through bureaucracy was completed.

Timman concieved mathematical engineering as distinguished in three levels:

1. applied mathematics to adequately describe the physical phenomena;
2. numerical analysis to convert the model into feasible algorithms;
3. programming and computation.

The result of such procedure would need to be evaluated by the applied mathematician, back at the first level. Both the applied mathematician and the numerical analyst were to be trained in Timman's mathematical engineering course.
The style of application he had in mind, was that of consultation: a dialogue of the engineer and the applied mathematician, viz. the mathematical engineer, resulting in a model of which the numerical handling was foreseeably feasible.

The clear break out of traditional Applied Mathematics is best illustrated when compared with earlier efforts to renew the paradigm from the inside. After Göttingen plea, by Felix Klein and Carl Runge, for Applied Mathematics as "Approximationsmathematik", Richard Von Mises was the first to cause subtle cracks in the concept of Applied Mathematics. According to the traditional concept results had thus far been judged for correctness purely by internal mathematical criteria —consistency, economy, sufficiency—, followed by a separate evaluation on adequacy. Von Mises in his famous opening article of the ZAMM in 1921 [Mises 1921] had proposed to judge the correctness of answers after the criteria of the engineer as well. And Timman radicalized this view:

> "Technology poses questions and *demands* an answer. Applied mathematics sometimes has no other way than to attack by "third degree" methods in order to get at the desired result, but it must realize that it may draw conclusions from results thus achieved only with utmost care.
> Where the extreme form of mathematical criticism fails, because it cannot do anything but to declare the lot of it unacceptable; then there must be different criteria based on experience in order to judge the one result trustworthy and the other untrustworthy. Such experience based criteria can only be acquired if the applied mathematician combines a comprehensive knowledge of the mathematical methods with a fundamental insight into the physical phenomena to which the mathematics is applied" [Timman 1952: p.15].

Trustworthyness for correctness: the essential shift in criteria mirrors the substitution of Applied

Mathematics by mathematical modelling, which was institutionalized in the discipline of mathematical engineering.
When in 1961, a few years after Delft, the Technical University of Eindhoven established its study course in Mathematical Engineering, the scope was widened inasmuch as to include operations research and management science. Thus the full rejoinder with the once declined Stevin- and Condorcet traditions was established.

5. MATHEMATICAL THOUGHT AS TECHNOLOGY
Simply by appearing as an engineering discipline mathematics might be claimed to have become technology. In this respect, however, Stevin did produce a precedent. Indeed, there was more to it. Before, parts of mathematics had spun off into engineering branches; mathematics had been delivering tools for other disciplines; mathematics had been serving as basic education for other ends. In all, various facets of mathematics had become technical in nature or had been placed in a technological perspective. This time, mathematical thinking as such appeared under a technological aspect. Such was the case, because the appeal to mathematical thought was now caught in a procedure, a procedure called mathematical modelling.

In the first place, the two sides of mathematical thought, mathematics at the service of external goals and mathematics for its own sake, had since Stevin split into diverging practices, indeed around 1800 into different disciplines. And within the purged science of mathematics Applied Mathematics stood for the faint memory of that other side. Cherishing the search for absolute truth, it definitely belonged to the practice of science for its own sake. Traditional Applied Mathematics could hardly be brought to bear upon technology other than from a distance, through its propaedeutic function. However, from the beginning of the twentieth century numerical methods began to spin off as almost technical tools of approximation.
How different was the far more pragmatic style of mathematical modelling, replacing Applied Mathematics. In the core of their work application-oriented mathematicians found themselves obeying the engineer's criteria, trustworthyness, instead of the sole pursuit of scientific truth.

As to the engineering disciplines, their silently assumed mathematization became gradually more explicit. Notably in the frontline disciplines of mechanical and aeronautical engineering the dynamical viewpoint of "applied mechanics" lead the way to recognizing mathematization as a fruitful procedure: mathematical modelling. The modern technology from the 1930's onwards proved quite mathematical in character, a point which the Delft mathematicians had trouble appreciating.
By consequence, mathematical engineering was not an accidental joint venture of the two fields, rather it represented an important step in the evolution of both, and a hallmark in their interrelation. A clearer expression of the interrelatedness of mathematical thought and engineering is hardly concievable.

As to mathematics, the other side of mathematical thought, viz. to serve external goals, turned out to be not a separate field, but an essential aspect of mathematics itself. The preparatory paper of a conference on training in applied mathematics, New York 1953, put it this way:

> "It is impossible to make a distinction between "pure" and "applied" mathematics on the basis of subject matter; any such attempt leads to absurdities. Applied mathematics is a matter of motivation, of attitude not aptitude, and the suggested alternative distinction between "monastic" and "secular" mathematics emphasizes the ultimate unity of mathematics" [Proceedings 1954: p.6].

Reinier Timman did not attend the conference, but John Tukey did and was inspired to develop the notion of mathematical engineering.

Reinier Timman 1917-1975
Picture from: *Symposium on applied mathematics dedicated to the late Prof.Dr. R. Timman* ed. by A.J. Hermans and M.W.C. Oosterveld. Delft: Delft U.P., 1978.

Around 1800 mathematics, pure and applied, had emancipated from Natural Philosophy. The result was a better insight in what mathematics really is. Around 1950, that purified science was reintegrated in the broader stream of mathematical thought as symbolized by mathematical modelling. Again our insight into what mathematics is, is enriched and recallibrated. The question as to what really is mathematics, or rather what is mathematical thought, remains unanswered. Gained is the view on another appearance of mathematical thought: as technology.

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