Centrum voor Wiskunde en Informatica

REPORT*RAPPORT*

MAS

Modelling, Analysis and Simulation

Modelling, Analysis and Simulation

Application of a Chimera Technique to the Computation of Subsonic and Transonic Bi-Airfoil Flows

L.C.Q. Yagua, B. Koren

REPORT MAS-E0407 MAY, 2004

CWI is the National Research Institute for Mathematics and Computer Science. It is sponsored by the Netherlands Organization for Scientific Research (NWO).
CWI is a founding member of ERCIM, the European Research Consortium for Informatics and Mathematics.

CWI's research has a theme-oriented structure and is grouped into four clusters. Listed below are the names of the clusters and in parentheses their acronyms.

Probability, Networks and Algorithms (PNA)

Software Engineering (SEN)

Modelling, Analysis and Simulation (MAS)

Information Systems (INS)

P.O. Box 94079, 1090 GB Amsterdam (NL)
Kruislaan 413, 1098 SJ Amsterdam (NL)
Telephone +31 20 592 9333
Telefax +31 20 592 4199

ISSN 1386-3703

Application of a Chimera Technique to the Computation of Subsonic and Transonic Bi-Airfoil Flows

ABSTRACT

This paper describes the application of a composite overset-grid technique to flow computations around a two-dimensional bi-plane. The flow is described by the steady, 2D, compressible Euler equations of gas dynamics. The discretization method used is a central finite-difference method with artificial dissipation. The overset-grid technique is of chimera-type and the bi-plane a bi-NACA0012 airfoil.

2000 Mathematics Subject Classification: 65N50; 65N55; 76H05

Keywords and Phrases: compressible flows; Euler equations of gas dynamics; mesh generation; domain decomposition; composite overset-grid (chimera) techniques

Note: The work was carried out under CWI-project MAS2.1 ``Computational Fluid Dynamics and Computational Electromagnetics''.

Application of a Chimera Technique to the Computation of Subsonic and Transonic Bi-Airfoil Flows

L.C.Q. Yagua and B. Koren
CWI
P.O. Box 94079, 1090 GB Amsterdam, The Netherlands

ABSTRACT
This paper describes the application of a composite overset-grid technique to flow computations around a two-dimensional bi-plane. The flow is described by the steady, 2D, compressible Euler equations of gas dynamics. The discretization method used is a central finite-difference method with artificial dissipation. The overset-grid technique is of chimera-type and the bi-plane a bi-NACA0012 airfoil.

2000 Mathematics Subject Classification: 65N50, 65N55, 76H05
Keywords and Phrases: compressible flows, Euler equations of gas dynamics, mesh generation, domain decomposition, composite overset-grid (chimera) techniques.
Note: The work was carried out under CWI-project MAS2.1 "Computational Fluid Dynamics and Computational Electromagnetics".

1. Introduction

Composite overset-grid techniques enable the generation of structured grids around complicated geometries and are particularly useful when one needs to rotate or translate one piece of geometry with respect to another. The first requirement to be fulfilled by an overset-grid technique is that the computational domain is entirely covered by grids. The overset-grid technique to be considered here is of chimera type [2, 8].

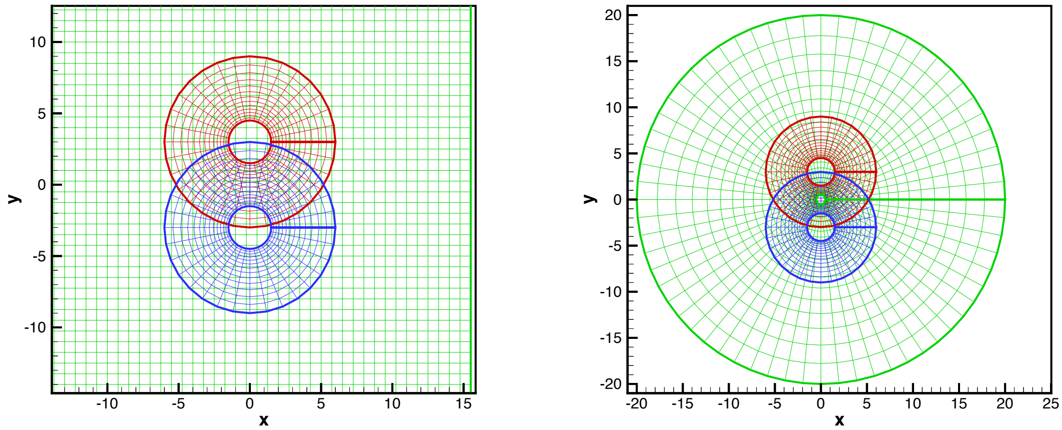
Overset-grid techniques give us the opportunity to adaptively refine a single subgrid without changing the other subgrids [6], and they can be combined with multigrid methods [4].

A chimera technique requires two specific additional algorithms: (i) an algorithm for generating the separate grids and for cutting holes in the grid parts which are overlapped, and (ii) an algorithm for interpolating solutions and possibly righthand sides between the various grids.

As mentioned, the test geometry to be considered is a bi-NACA0012 airfoil. For a precise definition of this geometry, we refer to [3]. The specific test cases to be considered are: (i) $M_\infty = 0.5$, $\alpha = 0$, (ii) $M_\infty = 0.75$, $\alpha = 0$ and (iii) $M_\infty = 0.55$, $\alpha = 6^o$. The first and the last test case are taken from [3].

As main grid for the chimera technique, i.e., as grid which reaches into the far field, we may choose between, e.g., a Cartesian or an O-type grid. As overlying subgrids we opt for two O-type grids, smoothly fitting around each of the two airfoils. If equidistant, a Cartesian main grid (Figure 1a) has the disadvantage that it is much too fine in the far field. By stretching the Cartesian grid, this can be improved, but then – locally – the grid spacing at far-field infinity is still too fine. We prefer a stretched O-type main grid (Figure 1b).

The contents of this report are the following. Section 2 briefly describes the chimera technique, and Section 3 the discretization method for the Euler equations. Numerical results are presented in Section 4 and conclusions are given in Section 5.



a. **b.**

Figure 1: Two types of main grid. **a.** Cartesian. **b.** O-type.

2. Chimera technique

In this section we mention the essential ingredients of the chimera technique: the grid generation, the grid-point elimination (hole cutting), and the solution interpolation.

2.1 Grid generation

We create a set of structured grids. One of these grids covers the entire computational domain, this is the main grid. All other grids will be subgrids. In Figures 1a and 1b, the large grid is the main grid and the two small grids are subgrids. The grids are generated independently of each other.

2.2 Grid-point elimination

Grid points will be eliminated in both the main grid and the subgrids. At first, points in the subgrids will be eliminated, namely all points which are outside the computational domain and all points which overlap with other subgrids and which are not needed for the solution (or righthand side) interpolation (Figure 2). Next, in the main grid all points are eliminated which are overlaid by subgrids and which are not needed for the interpolation. Doing so, in case of our bi-airfoil, a hole is created in the main grid (Figure 3a). A close-up of the composite grid is given in Figure 3b.

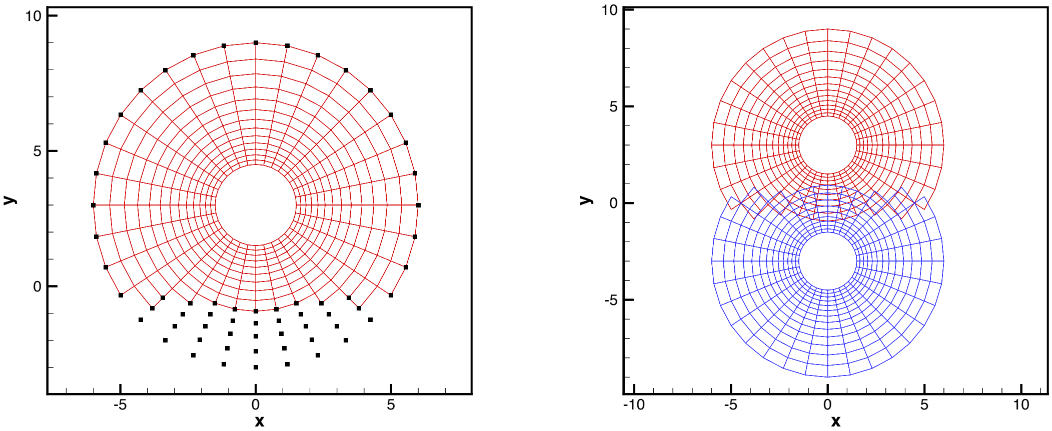
2.3 Solution interpolation

The interpolation applied for the solution (and possibly the righthand side) is bilinear, because of the good experiences with this in other two-dimensional problems [9, 10]. In case of more than one underlying grid, interpolation is done on the finest underlying grid.

3. Discretization method for the Euler equations

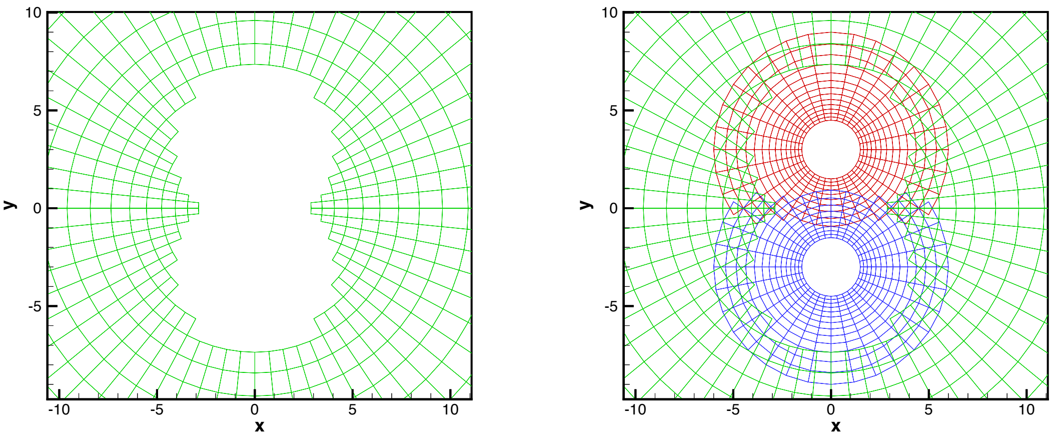
We consider the flow represented by the two-dimensional Euler equations written in general coordinates:

$$\frac{\partial \hat{Q}}{\partial \tau} + \frac{\partial \hat{E}}{\partial \xi} + \frac{\partial \hat{F}}{\partial \eta} = 0, \tag{3.1}$$



a. **b.**

Figure 2: Subgrids. **a.** All points in the upper subgrid that were eliminated, or that will be filled by interpolation. **b.** Both subgrids after the elimination process.



a. **b.**

Figure 3: Close-up of main grid. **a.** Without subgrids. **b.** With subgrids.

where the flux vectors $\hat{E}$ and $\hat{F}$ are defined by [7]

$$\hat{E} = J^{-1} \begin{pmatrix} \rho U \\ \rho u U + p\xi_x \\ \rho v U + p\xi_y \\ (e+p)U - p\xi_t \end{pmatrix}, \tag{3.2a}$$

$$\hat{F} = J^{-1} \begin{pmatrix} \rho V \\ \rho u V + p\eta_x \\ \rho v V + p\eta_y \\ (e+p)V - p\eta_t \end{pmatrix}, \tag{3.2b}$$

and $\hat{Q}$ by

$$\hat{Q} = J^{-1}Q = J^{-1}\left(\rho, \rho u, \rho v, e\right)^T, \tag{3.3}$$

where ρ is the density, u and v the Cartesian velocity components, p the pressure and e the total energy per unit of volume, and where the Jacobian J is defined by

$$J = \xi_x\eta_y - \xi_y\eta_x. \tag{3.4}$$

The variable e is defined by

$$e = \frac{p}{\gamma - 1} + \frac{1}{2}\rho\left(u^2 + v^2\right) \tag{3.5}$$

and the contravariant velocity components U and V by

$$U = \xi_t + \xi_x u + \xi_y v, \tag{3.6a}$$

$$V = \eta_t + \eta_x u + \eta_y v. \tag{3.6b}$$

The Euler equations are discretized by the central finite-difference approximation [1]. To solve the discretized equations, the explicit method

$$\hat{Q}^{n+1} - \hat{Q}^n = R_\xi + R_\eta \tag{3.7}$$

is used, with

$$R_\xi = -\Delta t \delta_\xi \hat{E}^n + D_\xi, \tag{3.8a}$$

$$R_\eta = -\Delta t \delta_\eta \hat{F}^n + D_\eta, \tag{3.8b}$$

where D_ξ and D_η are artificial dissipation terms defined by

$$D_\xi = -\Delta t \varepsilon_E J^{-1}\left(\nabla_\xi\Delta_\xi\right)^2 J\hat{Q}^n, \tag{3.9a}$$

$$D_\eta = -\Delta t \varepsilon_E J^{-1}\left(\nabla_\eta\Delta_\eta\right)^2 J\hat{Q}^n. \tag{3.9b}$$

In here δ_ξ and δ_η are central difference operators, and Δ and ∇ are forward and backward difference operators. The coefficient ε_E must be smaller than $\frac{1}{24}$ [7].
The operators δ, Δ and ∇ in the ξ-direction are defined by

$$\delta_\xi(\hat{E}^n)_{i,j} = \frac{1}{2}\left(\hat{E}^n_{i+1,j} - \hat{E}^n_{i-1,j}\right), \tag{3.10a}$$

$$\Delta_\xi(Q^n)_{i,j} = \left(Q^n_{i+1,j} - Q^n_{i,j}\right), \tag{3.10b}$$

$$\nabla_\xi(Q^n)_{i,j} = \left(Q^n_{i,j} - Q^n_{i-1,j}\right). \tag{3.10c}$$

The operators in the η-direction are defined in a similar way.

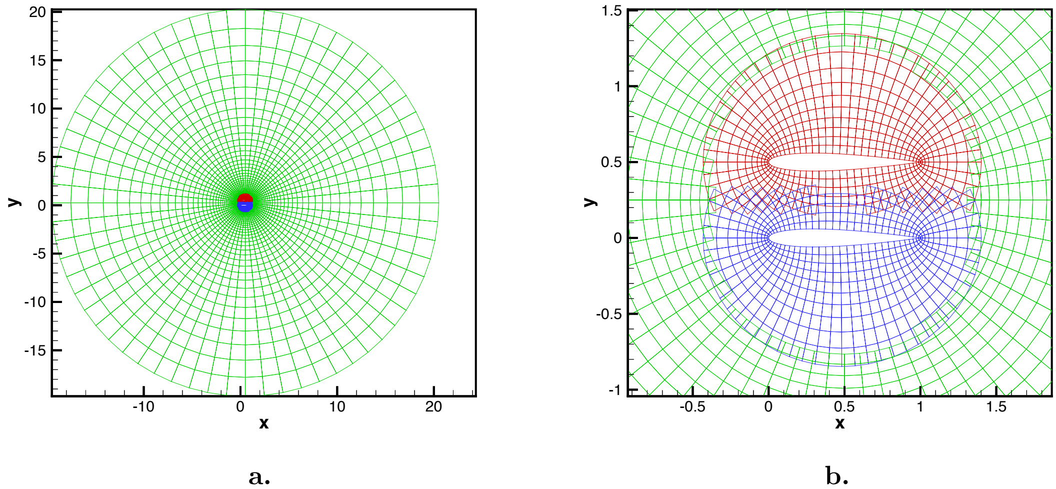


Figure 4: Composite grid for bi-NACA0012 airfoil. **a.** In full. **b.** In detail.

4. Numerical results

The subgrids fitted around the airfoils have 65 points in the circumferential direction (the ξ-direction) and 11 points at a maximum in the radial direction (the η-direction). The main grid has 65 points in ξ-direction and 41 points in η-direction. Graphs of the final composite grid are given in Figure 4.

We start by doing a numerical simulation with a free-stream Mach number $M_\infty = 0.5$ and angle of attack $\alpha = 0$. Numerical results are shown in Figures 5 and 6. As it should be, the solutions are perfectly symmetrical. From the Mach-number distributions (Figure 5), we learn that the solution is fully subsonic. Hence, the entropy distribution is a direct measure for the solution accuracy. Comparing the present relative errors (Figure 6b) with those from [5] for the comparable $M_\infty = 0.5$, $\alpha = 6^{\text{o}}$ case, it appears that for the equally fine grids applied here, the current error level is significantly lower. This is to be attributed to the fitting of both airfoils with O-type grids, instead of with poor H-type grids, as is done in [5].

We proceed by presenting numerical results for $M_\infty = 0.75$, $\alpha = 0$, a case not considered in [3]. The computed Mach-number distribution is presented in Figure 7. Note that this flow is transonic. We think that the minor asymmetry near the shock wave in between both airfoils is a plotting error due to the overlapping of the grids. The computed pressure distribution is given in Figure 8a and the relative entropy change in Figure 8b. At the airfoil's leading edges, this relative entropy variation is still a direct measure for the solution accuracy, it should be zero there. In the current results, the relative entropy error takes higher values than in the preceding subsonic case. A good explanation for this is still missing, we assume that the tuning of the artificial dissipation is worse than in the first test case. Surface-pressure distributions for both airfoils are given in Figure 9. Also here, note the good symmetry. The not yet perfect satisfaction of the Kutta condition, which manifests itself through the not yet smooth pressure distribution near the airfoil's trailing edges, is expected to be caused by the probably not yet optimally tuned artificial dissipation. Also, the mild overshoots near the shock waves are expected to be caused by this. In Figure 10, we still present velocity vectors in the overlapping grid regions.

The last test case to be considered is the case $M_\infty = 0.55$, $\alpha = 6^{\text{o}}$, a case also considered in [3]. The computed Mach-number distribution is shown in Figure 11. The agreement with most of the numerical results presented in [3] is quite poor for this test case. In the present results, no transonic

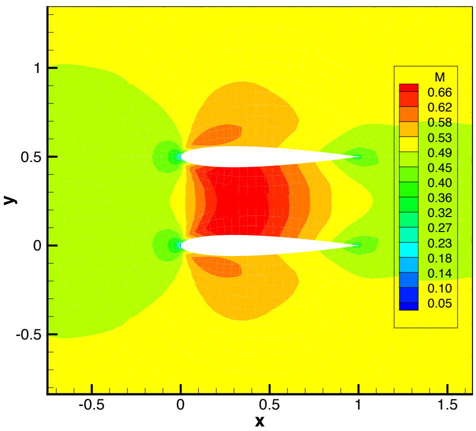
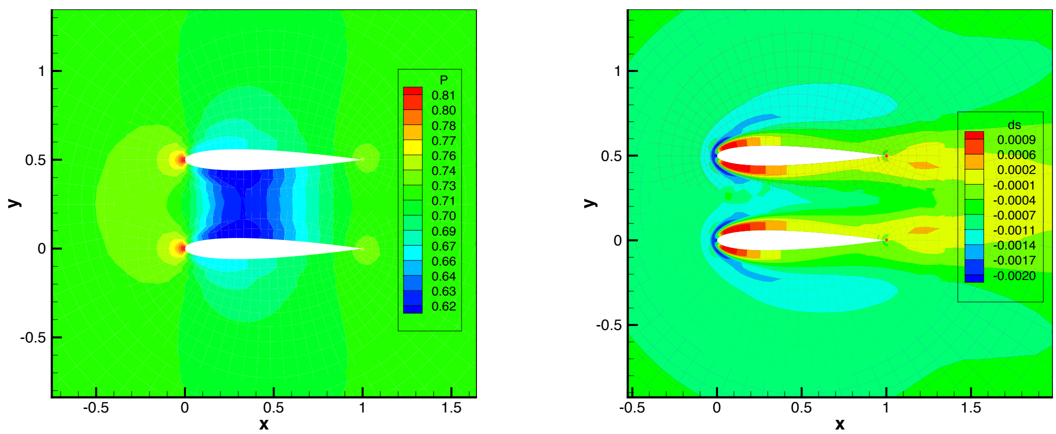


Figure 5: Mach-number distribution for the case $M_\infty = 0.5$, $\alpha = 0$.



a. **b.**

Figure 6: Iso-line distributions for the case $M_\infty = 0.5$, $\alpha = 0$. **a.** Pressure. **b.** Relative entropy change $\frac{s-s_\infty}{s_\infty}$, $s = \frac{p}{\rho^\gamma}$.

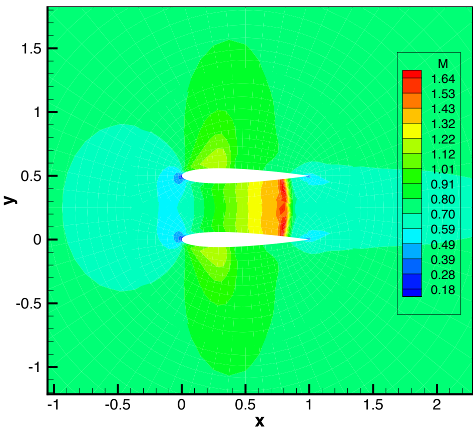


Figure 7: Mach-number distribution for the case $M_\infty = 0.75$, $\alpha = 0$.

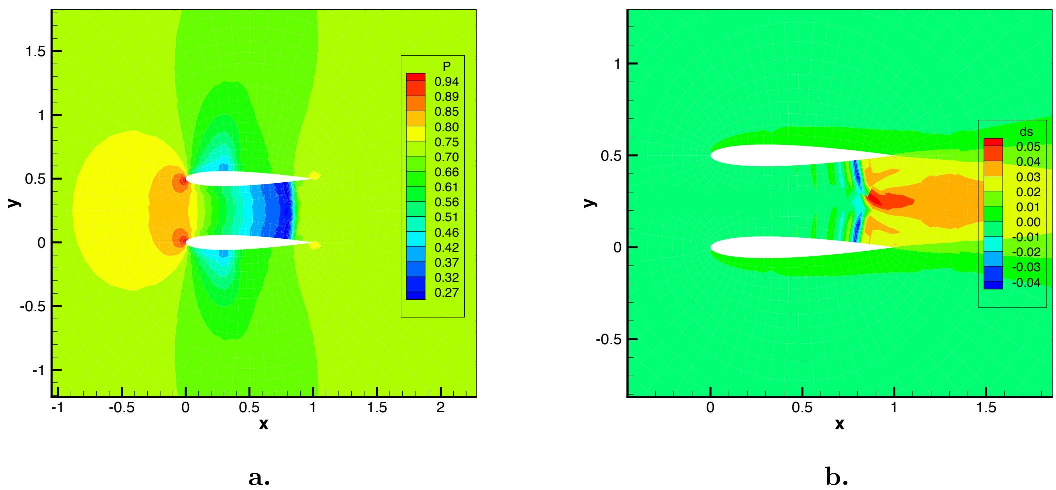
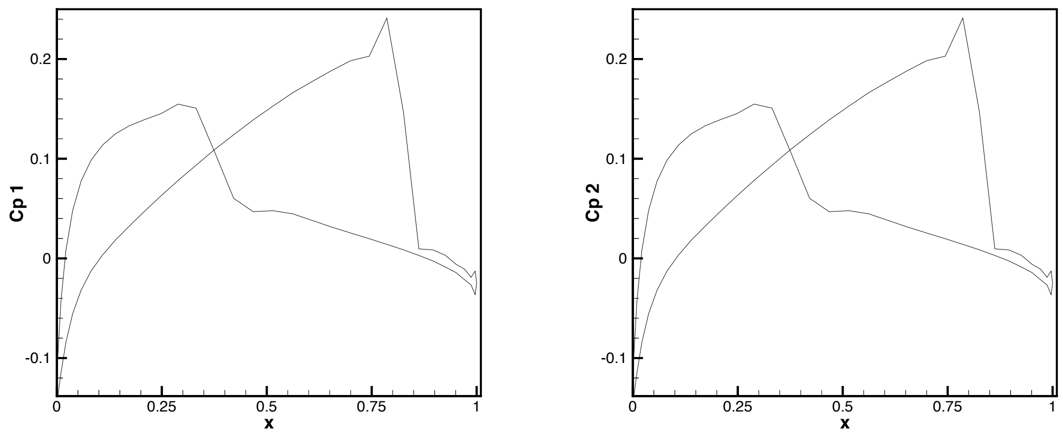


Figure 8: Iso-line distributions for the case $M_\infty = 0.75$, $\alpha = 0$. **a.** Pressure. **b.** Relative entropy change $\frac{s-s_\infty}{s_\infty}$, $s = \frac{p}{\rho^\gamma}$.



a. **b.**

Figure 9: Distribution of the surface pressure coefficient for the case $M_\infty = 0.75$, $\alpha = 0$. **a.** Along the lower airfoil. **b.** Along the upper airfoil.

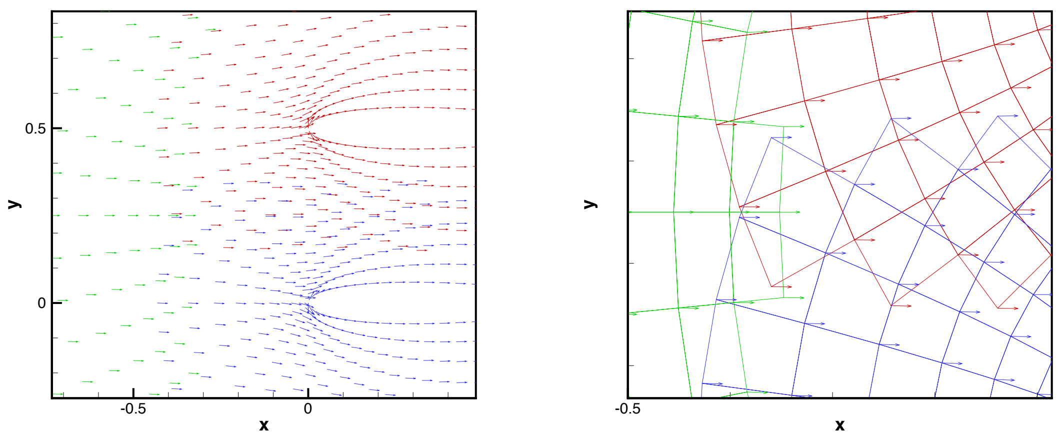


Figure 10: Close-ups of overlapping velocity-vector distributions for the case $M_\infty = 0.75$, $\alpha = 0$.

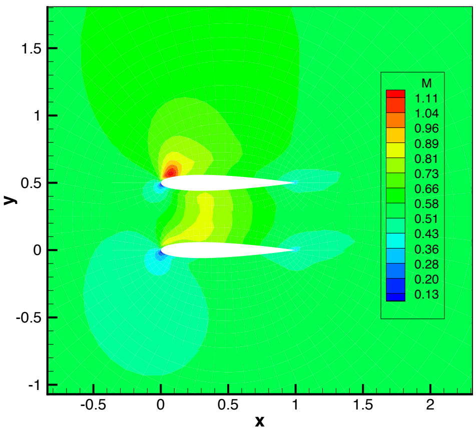
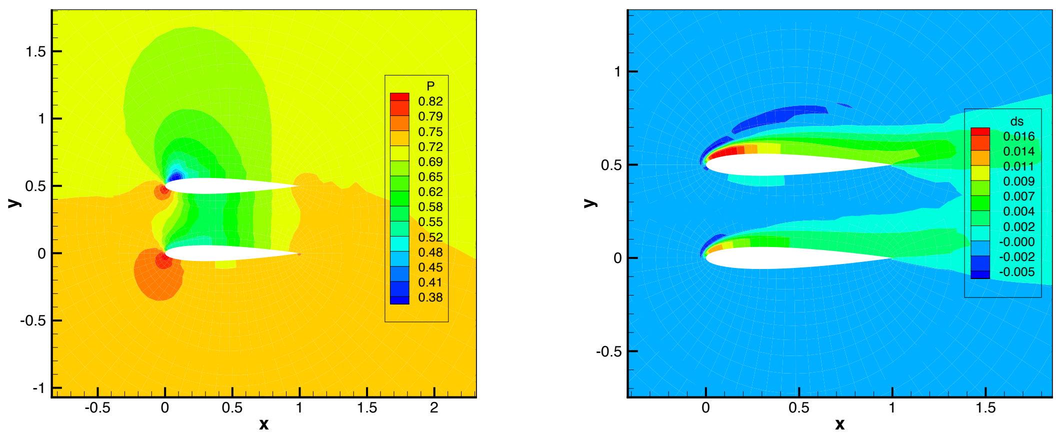


Figure 11: Mach-number distribution for the case $M_\infty = 0.55$, $\alpha = 6^o$.



a. **b.**

Figure 12: Iso-line distributions for the case $M_\infty = 0.55$, $\alpha = 6^o$. **a.** Pressure. **b.** Relative entropy change $\frac{s-s_\infty}{s_\infty}$, $s = \frac{p}{\rho^\gamma}$.

flow is obtained in between both airfoils, whereas in most reference results from [3] there is. The pressure and relative entropy error are given in Figure 12. The relative entropy-error plot (Figure 12b) shows quite high values near both leading edges. It is assumed that the artificial dissipation used for this test case is much too high.

5. Conclusions

The chimera technique appears to be a convenient overset-grid technique for the bi-NACA0012 airfoil as complex model geometry. The numerical solutions obtained appear to be quite smooth in the subregions where the subgrids overlap. Only in the case of a shock wave intersecting the region of overlap, we observed a small error (a slight distortion of the symmetry in the solution). In fact, the results are assumed to suffer from the poor spatial discretization of the Euler equations only: a second-order accurate central finite-difference method with explicitly added artificial dissipation.

References

1. Beam, R.M. and Warming, R.F., An implicit factored scheme for the compressible Navier-Stokes equations, *AIAA Journal*, **16**, 393–402 (1978).
2. Benek, J.A., Buning, P.G. and Steger, J.L., A 3-D chimera grid embedding technique, in *Proceedings of the Seventh AIAA Computational Fluid Dynamics Conference*, Cincinnati, OH, *AIAA-paper 85-1523*, (1985).
3. Dervieux, A., Van Leer, B., Périaux, J. and Rizzi, A. (eds.), *Proceedings of the GAMM-Workshop on the Numerical Simulation of Compressible Euler Flows*, Rocquencourt, 1986, *Notes on Numerical Fluid Mechanics*, **26**, Vieweg, Braunschweig (1989).
4. Fuchs, L., Calculation of flow using overlapping grids, in *Proceedings of the Eighth GAMM-Conference on Numerical Methods in Fluid Mechanics* (Wesseling, P., ed.), *Notes on Numerical Fluid Mechanics*, **29**, 138–147 (1990).
5. Hemker, P.W. and Koren, B., A non-linear multigrid method for the steady Euler equations, in *Proceedings of the GAMM-Workshop on the Numerical Simulation of Compressible Euler Flows*, Rocquencourt, 1986, *Notes on Numerical Fluid Mechanics*, **26**, 175–196 (Dervieux, A., Van Leer, B., Périaux, J. and Rizzi, A., eds.), Vieweg, Braunschweig (1989).
6. Henshaw, W.D. and Schwendeman, D.W., An adaptive numerical scheme for high-speed reactive flow on overlapping grids, *Journal of Computational Physics*, **191**, 420–447 (2003).
7. Pulliam, T.H. and Steger, J.L., Implicit finite-difference simulations of three-dimensional compressible flow, *AIAA Journal*, **18**, 159–167 (1980).
8. Steger, J.L., A viewpoint on discretization schemes for applied aerodynamic algorithms for complex configurations, in *Algorithmic Trends in Computational Fluid Dynamics*, 3–15 (Hussaini, M.Y., Kumar, A. and Salas, M.D.), Springer, New York (1993).
9. Yagua, L.C.Q. and Azevedo, J.L.F., Compressible flow simulation using overset multiblock grids, in *Proceedings of the Fourth Symposium on Overset Composite Grid and Solution Technology*, Aberdeen, MD (1998).
10. Yagua, L.C.Q. and Azevedo, J.L.F., Aplication de la tecnica chimera para la solucion de las Ecuaciones de Euler bidimensionales, *Revista de la Informacion Tecnologica*, **13**, 117–124 (2002).